

PotenciAT Model: Simulation of Urban Structure and Socioeconomic Evolution under a Complexity Science Approach

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Abstract

It is proposed a model for the urban sociospatial dynamics, considering transformation, either spatial or of urban land, as a result of the interaction among different levels of a system: individual decisions, urban subsystems and urban macro system, generating an ever changing process.

Growth and activity location follow principles such as attractiveness, accessibility, rent-gap, spatial interaction and social and economic interdependence.

Urban dynamic representation will occur from the model POTENCIAT based on activities which, at each interaction, allocate these activities in a system represented by regular cells. The cells occupied by complementary activities interact with the new ones, e.g., attraction locations. Those cells that contain incompatible activities will have negative values, causing repulsion.

The analogy used reduces the urban system to a field of positive and negative charges, simulating the attractiveness of each urban activity located spatially generating attraction and repulsion fields that affect new allocations.

Keywords:social and economic interdependence, spatial interaction, complex systems, urban dynamic, attractiveness.

Introduction

Concerning urban geography, the urban structure results from the decision on where to locate social and economic activities. Each decision is taken considering the existing urban structure and it limits the future decisions, by the use of space or the relations that are established, therefore assuming a systemic behavior.

The concept of spatial interaction offers a vast abstract model, in which the city is a changing force field, created by the variation of the peoples distribution, as well as of other activities that attract or repel themselves. The growth and location of activities follow principles such as attractiveness, accessibility (Wingo 1961), agglomeration economy and socioeconomic interdependence. It is considered that these principles are guiding elements to activities that seek location in an urban system

In the presented model, the concepts associated to the classical theories of the activities' location mentioned above are reviewed under the complexity science point of view. Therefore, is specially based on Haken's synergetic theory (1983) and its fundaments, like order parameter and enslaving process to describe and simulate the urban structure through the dynamics of spatial transformation and soil use.

The model works as an automaton cellular, where each allocated activity modifies the environment, influencing the next allocation decisions, and reallocation by means of its attractiveness or repulsion relations. The Urban space is represented by an orthogonal grill, with cells where the allocated activities simulate the several kinds of populations that act in the urban system.

Theoretical Approach

The urban structure changes according to the needs of society. These changes are identified on physical characteristics such as volumetry of buildings, intensity of urban occupation, and infrastructure. Each allocation decision activities consider the existing urban structure that limits the capability of decision other activities through the space occupation and the relations established assuming a systemic behavior.

Economic Theory

Since the classic models of the locational theory, such as Theory of Isolated Space (Von Thünen, 1826), Industrial Location Theory (Weber, 1909), Central Place Theory (Christaller's, 1933) and the models firstly developed in the 60's, urban modeling have been enhanced by the economy field. The assumptions that guide this approach are typical of the neoclassical economy, based on the paradigm of perfect competition, which leads to a greatest possible welfare related to the consumer theory. The space is conceived as an isotropic plain, where location differences are reduced to only one variable: CBD distance. Families decide possible combinations of three kinds of goods: compound goods, space use and CDB distance. The Satisfaction Individual is limited by the resources they possess, that is, the maximization of utility is restricted to its available resources. As CDB distance increases, there is a reduction in income offer per space unit and there is an enhancement of compound goods consume and increase of transportation expenses.

The principal limitation of the economic theory is that it is based on the idea that the system reaches a balance point, a perfect competition scenario ruled by offer and demand, a condition that is rare in the real world. Another limitation is the impossibility of dealing with external events and the imperfect rationality of the decision agents (SCHUMPETER, J. 1976). This theory does not consider the speculative nature and the income accumulation frequently sought by owners and construction industries.

1.2. Functionalistic Approach

The functionalistic approach is based on empiric observations on regularities that manifest across the spatial interdependence in the distribution of social and economic activities in the urban space. These interdependences appear from flow of people, commodities and information, among the several places of urban activities which are called spatial interaction. Spatial interaction models compare cities or urban areas to gravitational or electromagnetic force fields (concepts linked to Newton's Theory). Cities are formed by distinctive elements: individuals, mercadorias, informações that take place and move themselves through space, communicating, attracting and repelling each other mutually.

One of the first models formulated using this theory was the Lowry's (1964). The objective of this theory was to identify work locations and residential areas and the existing relations between these two variables. Other examples of this kind of models are Wilson's (1970) and Wingo's (1961). Spatial interaction models made it possible to explain the urban form, describing the spatial and socioeconomic interactions. Today, there are several transportation models that use principles from gravitational models to predict urban flows, showing realistic results and great application. The major criticism to this model highlights the lack of consideration regarding economic issues, such as land market and building constructions. According to Krafta(1995), the basis of most of the models of spatial interaction is the theory of random use, of general balance (work net and soil use) and more than a half of the models use Lowry's (1964) as a basis.

The approaches on Spatial Interaction and Economics Models are two complementary ways to analyze the urban spatial structure, mainly on a high level of aggregation. Both are based on economic relations and has the neoclassic economic theory as a starting point: the urban structure components interact with themselves, restricting the possible changes in each one, reaching, in this way the system equilibrium..

1.3. Self-Organization and Urban Development Dynamic

Since the 80's and 90's the field of geography has been applying the theory of self-organization to explain the behavior of the spatial, physical, urban and regional system. Complexity science sees the world as a collective of components that interact among themselves, assuming characteristics that are called emerging properties of systems.

According to Haken (1983), the system is ruled by a certain number of collective variables which form the “order-parameter”. During instability moments, many parameters coexist and compete among themselves until one of them wins and enslaves the system’s components through its movement. This process is called “enslaving-principle” and considers that complex systems generate reproductive relations among its components, enslaving its movement. Besides, in some moments, the competition can be solved by using the cooperation among its components. Portugali (1994) considers that in a city, the enslaving process is visible in cases like the invasion-repulsion, invasion-succession, as well as in the “gentrification” phenomenon (Smith, 1982), because individuals segregators prefer to live among neighbors of their kind. This process happens when choosing where they will work as well as in evaluating the place already used by them.

This situation suggests the existence of a socioeconomic and cultural urban mosaic where some order-parameters can attract the attention of individuals orienting the dynamics of the system’s transformation. This can be considered a “deterministic chaos” (Portugali, 1997), which appears when many individual parts are suddenly attracted by a small amount of attractors. This context may seem chaotic from a macroscopic view. Activities compete among themselves and only one Pattern is able to enslave the system and lead to a new order. Likewise, different groups take over determined spaces and, after competing for them, the winner maintains its order-parameter. From that, an identity of urban and regional space is established by modifying the city and/or region structure.

1.4. Attractiveness

The urban macrosystem, due to its complexity, have many buildings that possess functions generating attractiveness and, through that, the movement of people and goods that perform complementary activities (spatial interaction). the state of the urban system is perceived by the individuals with activity units that will act on it based on the available information about its general condition. This information is never complete, but it enables only a partial view of the system, giving a certain uncertainty concerning the obtained results.

The transformations occurred in urban subsystems originated from the allocation process of urban activities can be:

- **Strengthening:** when activities related to subsystems are attracted to the areas from which they belong results in a higher land use with the same kinds of activities;
- **Weakening:** when socioeconomic activities suffer the process of deactivation;
- **Movement:** when there is transition of socioeconomic activities to areas that offer more locational advantages to these functions or, on the other hand, they are expelled their previous places.

Since we consider the urban macrosystem as a result of the interaction among urban subsystems, either of the situations mentioned above would cause instability within the urban macrostructure, therefore modifying its spatial structure.

1.5. Competition and Urban Space Configuration Process

The urban space is heterogeneous concerning the available characteristics in order to enable urban growth and also to the resources necessary to develop certain socioeconomic activities. Among these we can name the physical factors, location related to the service net, public facilities and the transportation system technology that reflect in advantages of urban areas to the development of different activities. It is considered that the possibility of change would happen due to factors could destabilize its socioeconomic complementarity relations, causing a new adaptation of its components in order to adjust to this new established organization. The “winners”, that is, those that are able to obtain the necessary resources to their survival and reproduction will be selected, the others will be eliminated.

2. Potenciat Model (PotencialChange and Intra-Urban Attractiveness)

The aim of the “POTENCIAT” Model is to simulate urban structure’s development dynamics that emerges from spatial and land use remaking. This process constantly modifies the urban system through phenomena such as growth, competition for space, the enslaving-principle and socioeconomic relations.

In this context, the spatial interaction concept is crucial, because it considers the urban system like an attraction and repulsion field, simulating the attractiveness of each urban activity spatially located. Because of this, we decided to consider gravitational models and spatial interaction as a source of constant spatial transformation processes.

POTENCIAT is an Automaton Cellular, where each cell represents a space unit of urban soil. In this space, the activities are allocated based on the available information, which consists of:

- A)** Generating attractiveness by means of equal activities and their complementary;
- B)** Generating repulsion of activities with attractiveness incompatible;

The system's attractiveness is calculated considering other constraints such as: system's occupation, available space as well as accessibility (calculated using the system's occupied cells).

Each urban activity defines their complementarity and repulsion relations, as well as the capacity of expelling other activities from the system. These characteristics are present in the defining equation of attractiveness potential of each activity. It is important to notice that the urban structure evolves at each interaction of the model, without reaching a balance state as seen in traditional spatial interaction models. Due to this fact, the information used for allocations in the system changes constantly.

2.1. Attractiveness

In order to apply the concepts described here, we must define the kinds of activities that will be distributed along a cellular region, considering the “Newman-like” neighborhood relations. The activities, vary according to “kind” (qualitative variable), and also in number of “activity units” (quantitative variable). To calculate it, we consider that the polarization applied by each occupied space by a determined activity enhances with the number of activity units allocated there and in its neighborhood. In other words, the calculation is an exponential equation, used similarly to gravitational models. We consider that the polarization performed by each occupied point by a determined activity increases with the number of activity units allocated in the cell, and decreases by using an exponential equation, similarly to space Interaction above mentioned. We consider that the indexer is different to different land uses, since it is known, intuitively, that distance variation may influence every kind of urban activity differently.

This is the most basic evolution equation of the system applied to each cell and each activity individually, generating the first matrix of attractiveness potential.

$$P_{calc}a_{ij} = C_{ij}n / d_{ij}^k \quad (1)$$

Equation .1. is a polynomial function to calculate Attractiveness Potential of each cell calculated to each kind of agent individually and is based on P_{calc} (First calculation of the cell's Attractiveness Potential), C_{ij} (total number of activity units in the cell), d_{ij} (distance among the cell to which the Attractiveness Potential is being calculated to and the other cells in the system) and k (attractiveness force reduction constant)

Furthermore, in the module that calculates attractiveness for each activity of the system, there are elements that limit the attractiveness variation according to the system's occupation.

2.2 System's Saturation

The urban density is not homogeneous on the occupation surface. Due to this, we chose to define individually the saturation of each cell that initiates with a minimum value that are the same over the entire cellular space. As the available space in each cell decreases, so does the attractiveness value that may become negative when saturation exceeds the predefined number. The cell's individual saturation increases only after the cell be selected by means of a “probabilistic sample” for the saturation threshold growth. The cell regains enough attractiveness potential value to attract new activity units, because the relation between real occupation and maximum occupation is modified.

2.3. System's Maximum Saturation

The saturation of each cell is the result of the model's initial conditions and of the system's evolution. In spite of this, we can define a horizon from which the saturation will not increase. This value is the system's maximum saturation, that is, the maximum value of activity units that can be allocated in each cell.

2.4. Occupation Coefficient

As long as the cell is occupied, its space reduces. This will influence the cell's attractiveness by means of the occupation coefficient. When Saturation reaches its maximum, the cell will only regain attractiveness if enough activities are expelled so that the occupation coefficient is attractive again.

2.5. Accessibility

Here, the urban structure is considered as a result of the relations among urban activity units, spatial interaction processes, competition for space, the enslaving-principle, etc. Accessibility is a force that makes an activity unit chooses to place themselves closer downtown due to the need to reach the areas with more access to resources and facilities. This "situation quality" is called accessibility and may be the dominant factor to determine land use and its intensity. The Accessibility measure "Ac" modifies at each interaction and is defined according to the following procedure:

Occupation:

If " $C_{ij} > 0$ ", then " $C_{acessij} = 1$ ", if not " $C_{acessij} = 0$ "

$$Ac_{ij} = C_{acessij} + \sum C_{acessij} / d_{ij} n^2 \quad (5)$$

Equation .2. is the accessibility measure based only from the spatial distribution of occupied cells of the system's. It has different weights for each activity of the system, according to their need for proximity to downtown and its resources.

2.6. Final Individual Attractiveness Potential

$$Pot_{ij} = (P_{calc_{ij}} / q_{ij}) + (Ac_{ij} * \omega) \quad (6)$$

Equation .3. defines Pot_{ij} (each cell's final individual Potential) considering $P_{calc_{ij}}$ (Attractiveness Potential), q_{ij} (occupation coefficient of each cell), Ac_{ij} (Cell's Accessibility) and ω (accessibility Weight) which is different to each kind of activity, according to its needs.

By means of this procedure, a polarization continuous surface is generated from each activity in relation to itself. The calculation of the 1st potential of attractiveness matrix takes into consideration each activity that attracts its peers. The interaction among the different attractiveness matrixes relates the different activities that make the urban system according to their social and economic relations of complementarity.

2.7. Interaction

The technique used to model the system is based on mapping the existing attractiveness to each activity of the system. It is important to mention that the attractiveness is calculated to each kind of activity individually; besides, the same cell can present positive values to a determined agent and negative to another.

$$P_{ij} = \sum \omega P_{po} a_{ij}, \omega P_{po} b_{ij}, \omega P_{po} c_{ij}, \dots \omega P_{po} n_{ij} \quad (7)$$

Equation .4. is defined as P_{ij} , (potential attractiveness final of each activity) calculated from the interaction between the second matrixes of individually attractiveness. $P_{calca_{ij}}, P_{calcb_{ij}}, P_{calc_{ij}}, \dots, P_{calcn_{ij}}$ are Individual Attractiveness of each activity and ω is the Weight attributed to the other activities according to influence, positive or negative, over the activity to which the potential equation is being applied..

2.8. Growth

Exogenous growth tax defined to each activity will generate demand for space, so that there is urban sprawl and densification. Besides, there are the so-called multiplying activities, like Industry.

It is also considered the natural growth of dependent activities. In this situation the transportation system is not considered. It is supposed that the transportation costs are added to the equation with the attractiveness decrease related to distance.

2.9.Allocation

The system's central places emerge from successive interactions that modify the attractiveness of each cell in relation to the others. With this, there is a series of areas that have distinct sorts of attractiveness, calculated to each kind of activity. It is considered that the attractiveness of each cell denotes its potential for allocation of new activities.

Taking this into consideration, we can calculate the probability of occupancy of each cell from its attractiveness and available space. The outcome of the calculation probability will be multiplied by a random number between 0 and 1, generating a "probabilistic sample" that will define the activity units' location choice. This sample is fundamental to the model simulation process, for it allows, through the uncertainty of results, the possibility to innovate in terms of activities' allocation. This way, the attractiveness potential field does not define the activity units' allocation, but it serves as a "location guide" element to the next interactions.

2.10.Evaluation

The same cell can offer allocation advantages to different activities at different times. At each interaction the "evaluation" verifies which activities are found in areas of negative attractiveness potential. The expulsion can emerge from evolution equation of each activity that governs their ability to expel other incompatible activities that occupy the same space. The resulting values are multiplied by a random number between zero and 1, similarly to the choice of cells to allocate new activity units. just like the attractiveness case, the sample is fundamental for the model to work. Dislocated activity units in an interaction will be reallocated during posterior interactions. We also define a minimum limit from which all the activities in one cell that has negative potential should be removed. As we can see all the variables of the system are related and depends one to each other. Next, we see the general diagram that explains how the model works, where we can see the calculation sequence.

3. "Potenciat Model" Simulation

Here we present the simulation of a 50X50 cell-system, whose basic activity is the Industry and dependent activities are residential and commercial. The selected initial cell was the one located at line 25, column 25 (25,25) from the cellular territory. To each activity we define differentiated behaviors as seen in the equations that define the Attractiveness Potential Area, shown below. The occupation coefficient is differentiated to industry, that needs more space, as well as to "Population 1", that prefers more space. To business, accessibility is defined as principal variable. Accessibility is also important to Populations 2 and 3 which who give up of the living space for greater accessibility differentially of "Population 1". Another important aspect is that we tried to attribute less weight as possible to each activity in relation to itself in the attractiveness potential equations. This definition aims to verify the possibility to form groups without indication directly in the system's evolution equation. We consider the preference of each activity in allocating itself near its peers, but groups emerge because activities alike have the same kind of location preference.

In the Beginning of Simulation, the attractiveness field is mainly formed by one factor: the system's accessibility. The location of only one basic activity agent creates a polarization due to the accessibility value that generates the attractiveness potential values for other activities. At this simulation's initial stage, accessibility is the only element that defines attractiveness because of the inexistence of other factors that will appear during simulation with the allocation of the other activities of the system.

As the cells are occupied, we have individual saturation changes and, therefore, coefficient changes too. In the end of the Simulation there is greater difference among the activities locations. The areas already occupied by the other activities start to become negative to the industry, which starts to dislocate to the borders. Commerce activity is affected by accessibility defined as a combination of easy access and nearness required to optimize the economic activity. "Population 1", which was initially located at the system's most central area, starts to locate at the "border" and forming specialized clusters.

This behavior occurs because "Population 1" is "expelled" from the most central areas, due to its "preference for bigger places" defined in the equation that calculates the occupation coefficient for this activity. Accessibility is more important than space for population 2, and therefore it goes back to a more concentrated occupation in some places at the central area. Population 3 ends up occupying the system's outskirts due to its little or no capacity of expulsion relative to other activities, as defined in its attractiveness potential equation. Satisfaction is limited by the resources they possess.

4. Results Evaluation

The simulation resulting of the "POTENCIAT Model" will be evaluated for their intrinsic qualities are brought to light. The aim is to reveal to what extent the model reproduces real-world urban processes that cannot be detected by simple observation of concentration of activities. With this objective, the results of "POTENCIAT Model" will be compared with the city of Curitiba through different techniques which results are presented here.

4.1. Spatial Statistic

In this paper, we used a specific technique of spatial statistics to detect the presence of spatial autocorrelation of activities considered in the simulations of the POTENCIAT model. This methodology allows us to investigate whether there are "spillovers" between clusters, or in other words, if the location of activities "overflows" to the development of the same activity in neighboring cells. This property was investigated using software OpenGeoDa 0.9.8.14 (2009) applied on the Simulation here presented. This exercise has been developed in order to identify properties of spatial dependence and formation of groups each taking into account the total system and then each activity separately. As we can see, the structure formation occurs through a process of succession of the most significant groups, that with the system growth, leaves the central region towards the periphery. Becomes clearer here the issue of "spillover" cited above, since the main groups following clearly, the spatial relationship to the previous configuration. Thus we see that the internal motion of the areas of predominance of each activity and complements the data of the order parameter belonging to the previous analysis.

In a Real System, these transformations come from decisions of the different types of individual actors through important principles, like spatial interactions, socioeconomic relations, attractiveness, enslaving-principle and Synergy. This kind of analysis can detect the acting forces in each of these areas and the intensity reached by each of them. From the concepts developed (the order-parameter and the enslaving-principle, applied to the processes that form occupation patterns of urban spaces) it would be possible to measure the change potential of land use and of spatial change in these areas. Finally, this work aimed to show a brief discussion that suggests the possibility of constructing nonlinear social and economic models in the light of nonequilibrium paradigm to understand and describe the urban structure.

4.2. Fractal Geometry

Many phenomena and forms found in nature cannot be explained in the mold of conventional mathematics, which will require one special way math that is able to explain morphologic aspects. One way to achieve this result is to use the mathematics of fractals. Currently the fractal geometry, and in particular the fractal dimension, has been used in several areas of study of chaotic systems as characterization of objects, analysis and pattern recognition in images, texture analysis and measurement of length curves. For fractals, rather than occurs with Euclidean objects "perfect", each object has its own dimension. The irregular curves have a dimension of between one and two so that an uneven surface has a dimension between two and three(FRANKHAUSER, 1998). The application of fractal dimension will be in the Brazilian city of Curitiba in order to compare the results of real systems with simulated in POTENCIAT Model. For the application of fractal dimension in this work was used Fractalyse software developed by the group "City, mobility, territory" at the research center Thema coordinated research and Cécile Pierre FrankhauserTannier.

4.2.3 External Limits

Curitiba is the capital of the state of Paraná, and was founded in 1693, from a small village Scout. The city became an important stop shopping with the opening of the road between Sorocaba and Viamão and has always been the target of major urban plans. Its morphologic behavior demonstrates homogeneity on the use of space along the iterations.

Being such a measure that detects the "roughness of a picture", low values of fractal dimension indicate regular shapes. Likewise, this behavior appears homogeneous on Simulation of POTENTIAT Model in form of space occupation along the iterations. Being such a measure that detects the "roughness of a picture", low values of fractal dimension indicate regular shapes.

4.2.4 Land Use

Curitiba: The shape of the occupation of each land use presented here comes from the historical survey of each district that indicates the activities located e each income level of its inhabitants. This information came and Historical Maps of 1900, 1939, 1967 and 2000. The value of fractal dimension when applied over the outer edge of Curitiba and Simulation 1 as a whole tended to decrease over time showing stabilization with respect to the outer edge of systems, both real and simulated. On the other hand, the observation of individual activities and their values allows us to say that, as in the case study, the interactions between different activities tend to increase the value of the fractal dimension of their occupations over time.

This feature is an emergent property of the system that arises due to the interaction of the continuous activities and the way they influence each other. The Urban Structure emerged as a result of the interplay of positive and negative feedbacks. The mathematical model here presented seeks to describe and simulate empiric phenomena observed in urban dynamic development and transformation. Activities have to compete on the urban field in order to occupy the most attractive area to them and this competition changes spatial configuration and relationships and also the condition of the spatial distribution of activities.

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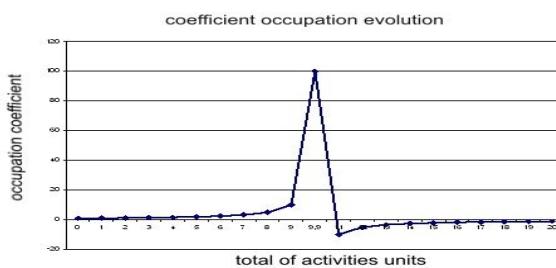
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6. Graphics, Tables and Figures

6.1. Graphic



Graphic 1.

Evolution of a cell's Occupation Coefficient with sat=10

6.2 Tables

Table 1. Simulation: Growth equations of each agent:

Activity units:	Growth	Tax
Industry: considered as basic activity	ind=toti*txind/100	2%
Business: Represents facilities in general	(res+res2+res3)*txcom/100;	20%
Population 1: high level of income	res=totp*txres/100 + ind*10,0	1%
Population 2: medium level of income	res2=totp2*txres/100 + ind*10.0	1%
Population 3: low level of income	res3=totp3*txres/100 + ind*10.0	1%

Simulation: Equations to Define the Attractiveness Potential Area:

Activity units	Attractiveness Potential Equations
Industry	$(0 * \text{indaux}[i][j] + 0 * \text{comaux}[i][j] - 0.1 * \text{popaux}[i][j] - 0.1 * \text{pop2aux}[i][j] + 0.1 * \text{pop3aux}[i][j] + 0.8 * \text{Access}[i][j]) / (\text{coeff}[i][j] * 10)$
Business	$(0 * \text{popaux}[i][j] + 0 * \text{pop2aux}[i][j] + 0 * \text{pop3aux}[i][j] + 0.0 * \text{comaux}[i][j] - 0 * \text{indaux}[i][j] + \text{Access}[i][j]) / \text{coeff}[i][j]$
Population 1	$(0 * \text{popaux}[i][j] - 0 * \text{pop2aux}[i][j] - 0 * \text{pop3aux}[i][j] + 0 * \text{comaux}[i][j] - 0.1 * \text{indaux}[i][j] + \text{Access}[i][j]) / (\text{coeff}[i][j] * 2)$
Population 2	$(0 * \text{pop2aux}[i][j] - 0.1 * \text{popaux}[i][j] + 0 * \text{pop3aux}[i][j] + 0 * \text{comaux}[i][j] - 0.1 * \text{indaux}[i][j] + 1 * \text{Access}[i][j]) / \text{coeff}[i][j]$
Population 3	$(0 * \text{indaux}[i][j] + 1 * \text{comaux}[i][j] - 0.1 * \text{popaux}[i][j] - 0.1 * \text{pop2aux}[i][j] + 0.1 * \text{pop3aux}[i][j] + 1 * \text{Access}[i][j]) / (\text{coeff}[i][j])$

Curitiba: morphology and fractal dimension evolution

YEAR	FRACTAL DIMENSION						
1900	1,09	1939	1,36	1967	1,319	2000	1,085

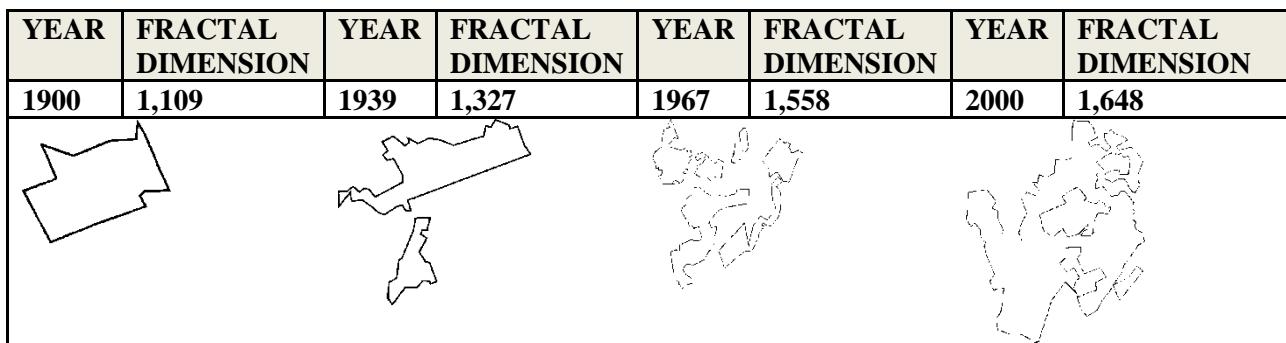
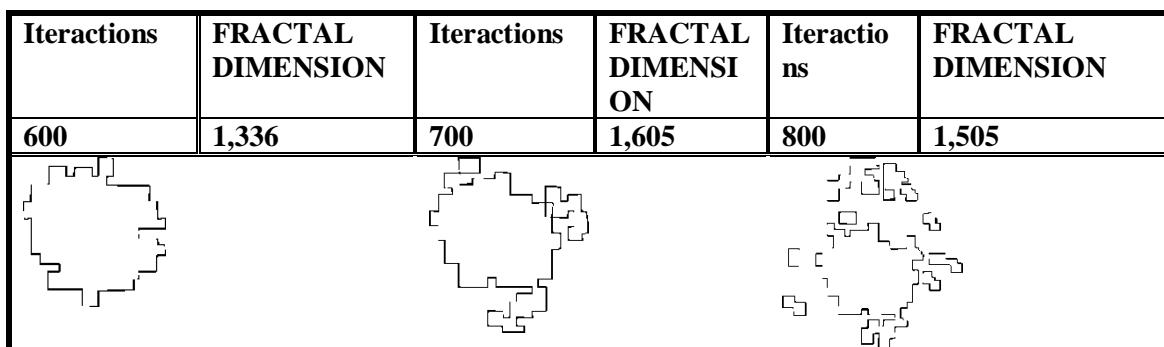
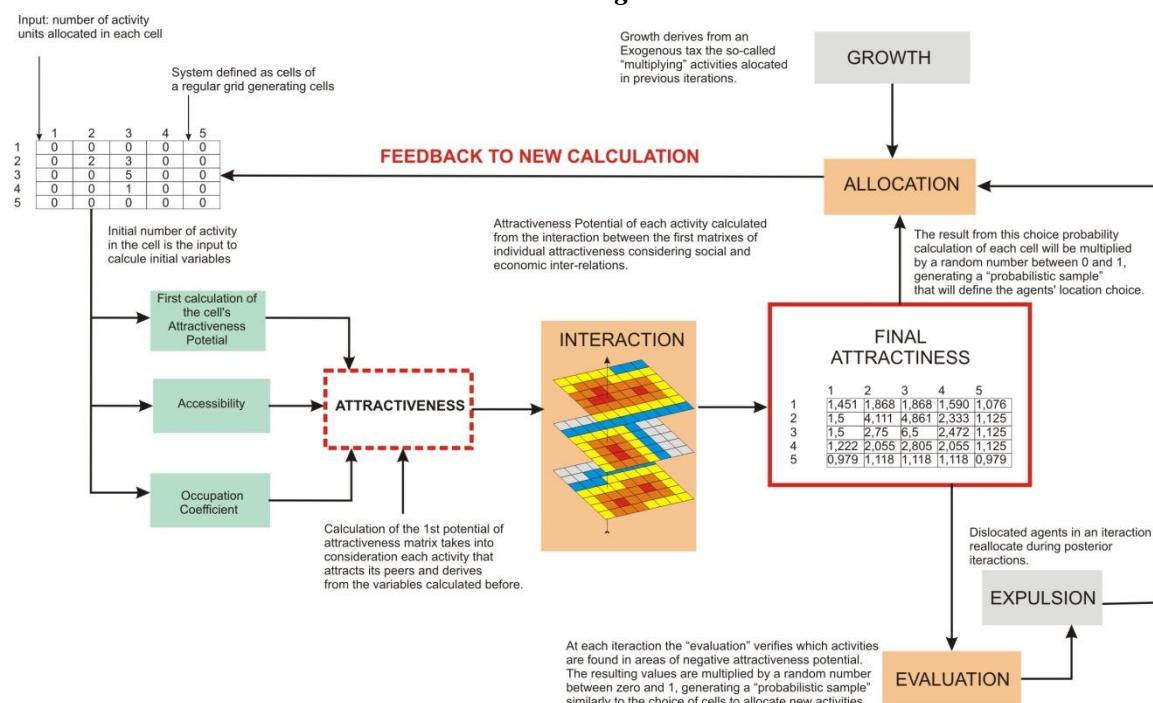
Curitiba Industry: occupation limits*Simulated System Industry: occupation limits***6.3 Figures****Figure 1. General Diagram of the Model's functioning:****6.1 Evolution of the Simulated System**



Figure 2. Initial Location, Accessibility Attractiveness Potential – Population 1 Activity



Figure3. 1- Calculated Accessibility, Occupation Coefficient and Total Occupation - 100 interactions

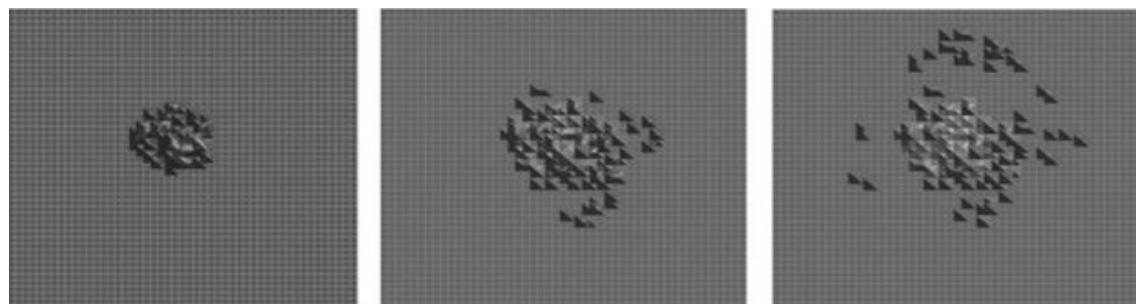


Figure4. Location of Industrial Activity – 500, 700, 800 interactions

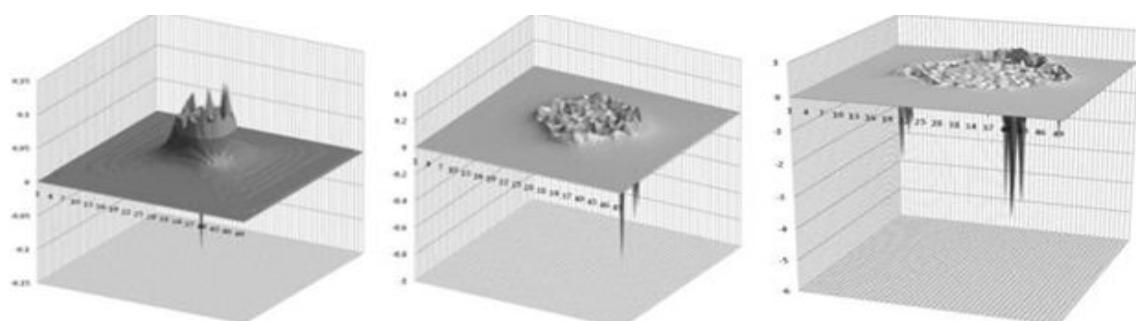


Figure 5. Attractiveness Potential of Industrial Activity 500, 700, 800 interactions

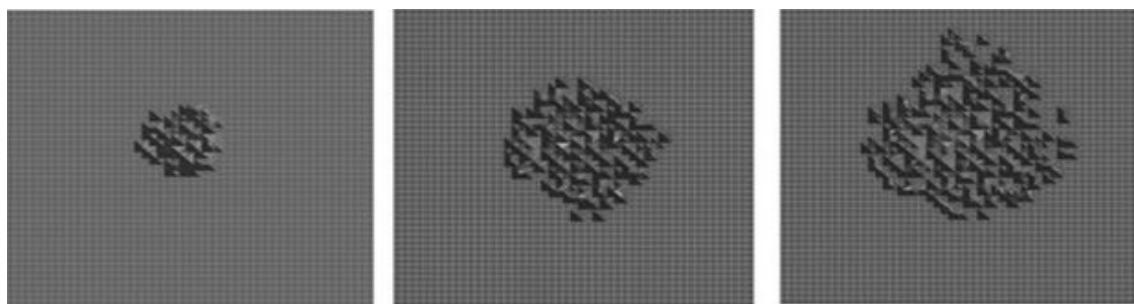


Figure 6. Location of Commerce Activity – 500, 700, 800 interactions

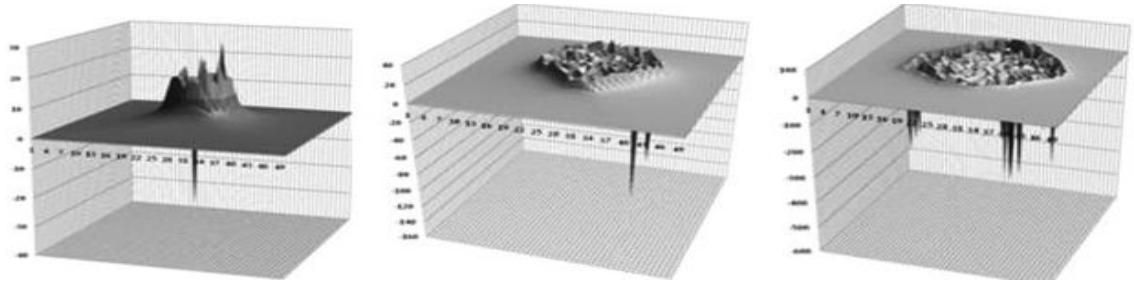


Figure 7. Attractiveness Potential of Commerce Activity 500, 700, 800 interactions

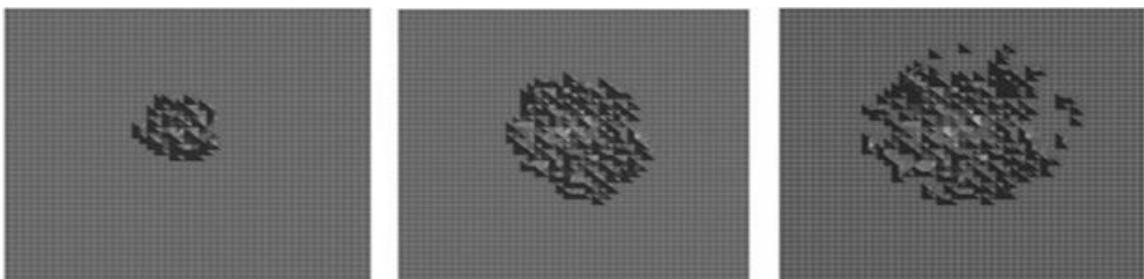


Figure 8. Location of Population 1 Activity – 500, 700, 800 interactions

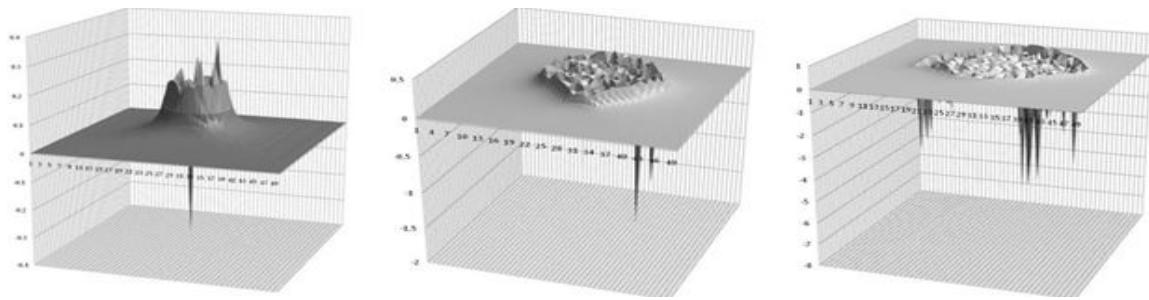


Figure 9. Attractiveness Potential of Population 1 Activity 500, 700, 800 interactions

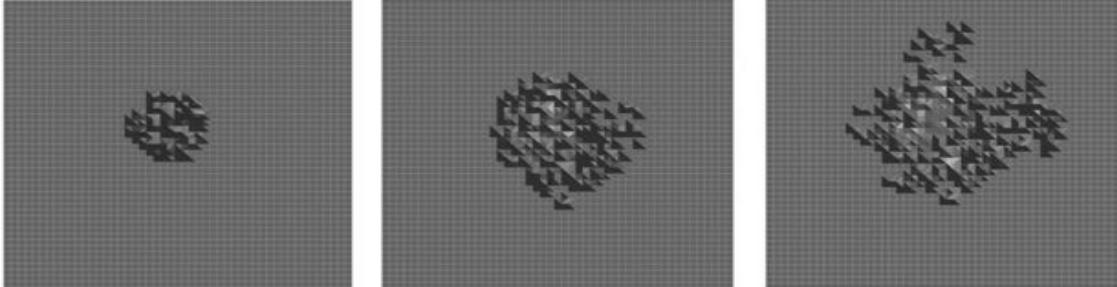


Figure 10. Location of Population 2 Activity – 500, 700, 800 interaction

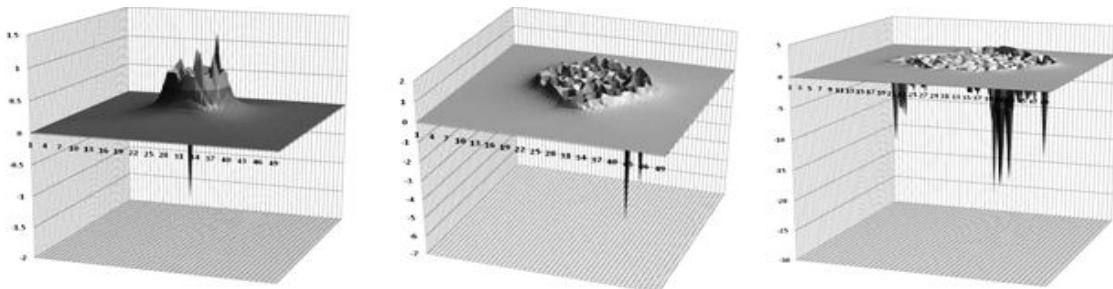


Figure 11. Attractiveness Potential of Population 2 Activity 500, 700, 800 interactions

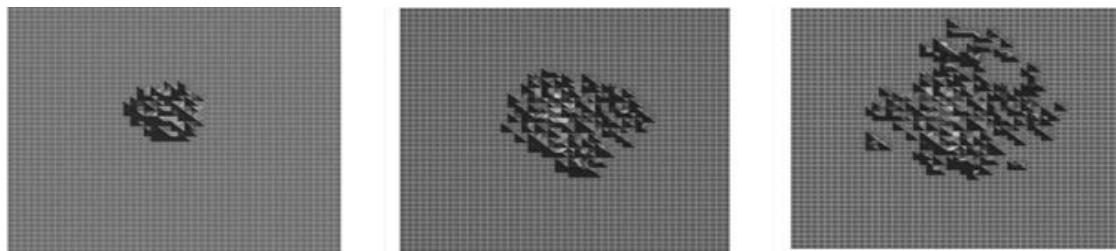


Figure 12. Location of Population 3 Activity – 500, 700, 800 interactions

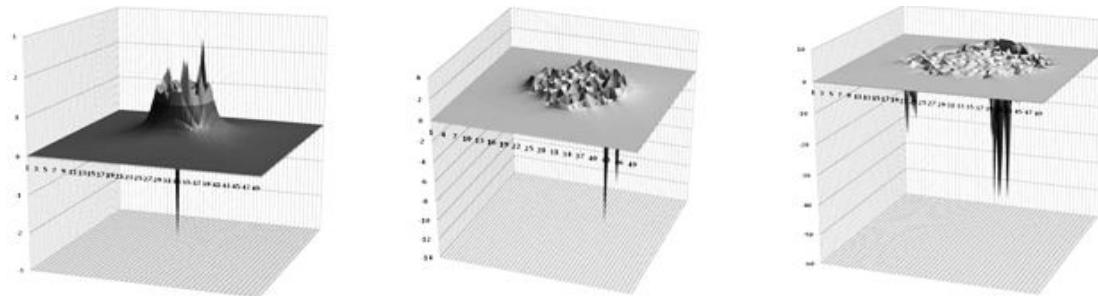


Figure 13. Attractiveness Potential of Population 3 Activity 500, 700, 800 interactions

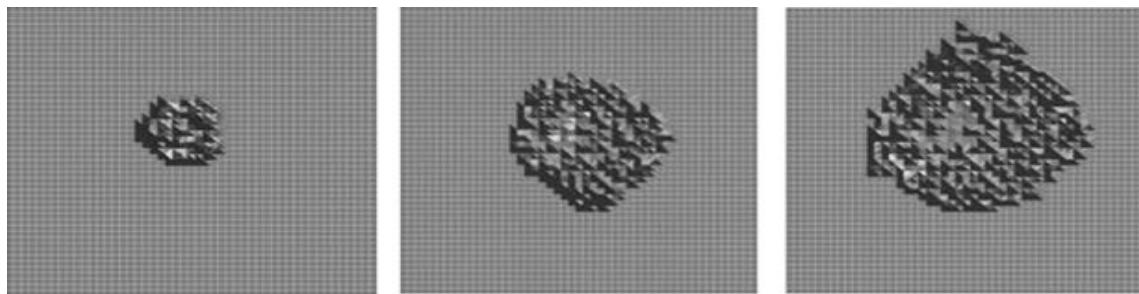


Figure 14. Location of Total System – 500, 700, 800 interactions

6.2 Evaluation of the results

1.2.3 Studied Areas in Spatial Statistic



Figure 14. Total System – 600, 700, 800 interactions



Figure 15. Industry – 600, 700, 800 interactions

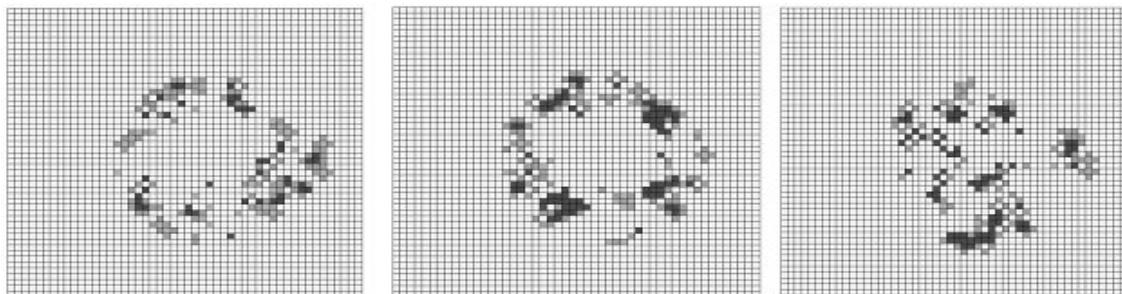


Figure 15. Commerce, Population 1 and 2 – 800 interactions